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INTRODUCTION

The purpose of the SPAN® appendix is to provide information about how to process risk array calculation for end of day and intra-day calculation.

In addition, this document describes the differences between the new release (from the 30th March 2012) corresponding to UCS V9 & SPAN® RMC and the previous one (before the 30th March 2012) corresponding to UCSV5 & SPAN®.

Firstly, you will find a summary of the changes, then, in the formulas, the detailed of the changes highlighted in yellow.

Please note that the main principles of the pricing models remain unchanged and that the differences are relying on how inputs data are used within these formulas.
SUMMARY OF CHANGES

Lot size
For calculation purpose, the lot size used in the calculation was the one of the contract in the previous release whereas in the new release, it is the lot size of the series.

Interest rate
The interest rate (including foreign interest rate when exist) is no longer converted to natural logarithm (Napierian logarithm) of the calculation for continuous models as Black 76 model and Garman Kohlhaagen model.
For discrete models such as Cox Ross Rubinstein (European and American style), there is no impact.

Time to maturity
For each scenario and to calculate the delta, one day in fraction of year is deducted from the time to maturity of the option. So, in comparison to the previous version, it is required to decrement life time by 0.00274 (please refer to the formulas below).

Number of iteration for Cox Ross modeling
For discrete models such as Cox Ross Rubinstein (European and American style), there is a number of iteration which is fixed to 30 whatever the model. This is a significant modification compared to the previous situation where according to the type of pricing model (American: n=7; European: n=30), an average was done between n and (n+1) iterations. In this new version there is no more average. The 30 iterations have just to be taken into account alone.

<table>
<thead>
<tr>
<th>UCS V9 &amp; SPAN® RMC</th>
<th>UCSV5 &amp; SPAN®</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1 - Contract value factor (standing for lot size)</strong></td>
<td><strong>The Contract Value Factor is collected at contract level</strong></td>
</tr>
<tr>
<td>a. There is no Contract Value Factor at contract level in SPAN® RMC. One contract value factor is selected from SPAN® file record 83 and set as series contract value factor.</td>
<td>a. The Contract Value Factor is determined at contract level even if it exists at series level.</td>
</tr>
<tr>
<td>b. So for the whole series, the Contract Value Factor will be unique.</td>
<td>b. For calculation purpose, it is the value at contract level which is used.</td>
</tr>
<tr>
<td><strong>2 - Time to maturity</strong></td>
<td>The time to maturity of each option is taken into account for the scenario pricing.</td>
</tr>
<tr>
<td>The time to maturity of the option series is decremented by one calendar day (1/365 = 0.00274) for the scenario pricing. So, T = Time to maturity – 0.00274</td>
<td>So, T = Time to maturity</td>
</tr>
<tr>
<td><strong>3 - Interest rate</strong></td>
<td>For calculation: option on futures contracts, options on index and currency options:</td>
</tr>
<tr>
<td>The value is loaded directly from UCS (5 firsts digits) an used without any conversion</td>
<td>- Rd : Yearly continuous financing domestic rate on the time to maturity of the option ( rd = Ln(1+ Rd) where Rd is the yearly financing domestic rate)</td>
</tr>
<tr>
<td>- RF : Yearly continuous financing foreign rate on the time to maturity of the option</td>
<td>- RF : Yearly continuous financing foreign rate on the time to maturity of the option ( rf = Ln(1+ Rf) where Rf is the yearly financing foreign rate)</td>
</tr>
<tr>
<td>For option on stocks: no gap.</td>
<td>For option on stocks: no gap.</td>
</tr>
<tr>
<td><strong>4 - Number of iteration</strong></td>
<td>The number of iteration is a Cox Ross Rubinstein input parameter</td>
</tr>
<tr>
<td>The number of iteration is no longer depending on the contract. A parameter is set as default whatever the contract:</td>
<td>The number of iteration depends on the contract:</td>
</tr>
<tr>
<td>European and American type Cox Ross Rubinstein iteration = 30</td>
<td>a. European type Cox Ross Rubinstein iteration = 30</td>
</tr>
<tr>
<td></td>
<td>b. American type Cox Ross Rubinstein iteration = 7</td>
</tr>
</tbody>
</table>
CHAPTER I

CALCULATION OF RISK ARRAY VALUES
A risk array can be defined as a set of numbers:

- For a particular contract
- At a particular point in time
- To be margined for a particular business function
- For a particular account.

Each risk array value specifies how a single long or short position will lose or gain value if the corresponding risk scenario occurs over the specified look-ahead time. The look-ahead time is the average time per day. So it reflects the unit of time into the future from the current time. By convention, losses for long positions are expressed as positive numbers, and gains as negative numbers.

**PRINCIPLE USED: MARKET PREMIUM AND THEORETICAL PREMIUM VALUATION**

1) **Option value**

1.1) **For end-of-day**: Option is valued using as inputs:

   a. Market price (or premium) value of the option product (converted to option currency if needed and scaled to the option size):
   
   \[(\text{Market price} \times \text{option CVF})\]

1.2) **For intra-day**: Theoretical option value is calculated using as inputs:

   a. Value of the underlying product (converted to option currency if need be and scaled to the option size):
   
   \[(\text{underlying price} \times \text{underlying CVF}) \times \text{Currency conversion rate} \times \text{option scaling factor.}\]
   
   \[\text{(Option scaling factor is equal to option CVF/underlying CVF)}\]

   b. Option implied volatility
   
   c. Option strike value:
   
   \[\text{(strike} \times \text{option CVF)}\]
   
   d. Time to maturity
   
   e. Interest rate
   
   f. Dividends

2) **Risk array theoretical option value**

   For each risk array point new theoretical value is calculated using as inputs:

   a. Value of the underlying product (converted to option currency if need be and scaled appropriately if CVF for underlying and CVF for options are different):
   
   \[(\text{underlying price} \times \text{underlying CVF} + \text{price scan range} \times \text{fraction}) \times \text{currency conversion rate} \times \text{option scaling factor.}\]

   b. Option implied volatility + volatility scan range * fraction
   
   c. Option strike value:
   
   \[\text{(strike} \times \text{option CVF)}\]

   d. Time to maturity – move ahead time
   
   e. Interest rate
   
   f. Dividends

3) **Risk array value**

3.1) **For end-of-day**: For each risk array point difference between valued market price and theoretical option value is taken as risk array value.

3.2) **For intra-day**: For each risk array point difference between theoretical value calculated for this point and theoretical option value is taken as risk array value.

From this description it is clear that:

- Risk array values will always come out appropriately scaled (in the option value terms).
- Price scan range as input in this algorithm has to be specified in underlying value terms.
CHAPTER II

VALUATION FORMULA FOR OPTIONS ON FUTURES AND INDICES
For determining the initial margin required to cover the positions on options on futures contracts (commodity for instance) and options on index, LCH.Clearnet SA uses a theoretical premium valuation formula of the type Black 76.

**ALGORITHM USED: THEORETICAL PREMIUM AND DELTA**

1. **Notation**

The algorithm requires the following data:

- \( U \) : Price of the underlying futures contract of the option
- \( E \) : Strike price of the option
- \( T_0 \) : Time to maturity of the option (in fraction of year (base: 365 d or 366 d)).
- \( T \): A time to maturity minus the "look-ahead time" of the option. The look-ahead time is average time per day (base: \((1/365) d \) or \(0.00274\)). So it reflects the unit of time (in fraction of year) into the future from the current time \((T = T_0 - \text{moveahead} or T = T_0 - 0.00274)\)
- \( \nu \) : Yearly volatility of the underlying (expressed in %).
- \( R \) : Yearly continuous financing rate on the life span of the option
- \( N(x) \) : Distribution function of the normal law
- \( \ln \) : Napierian logarithm
- \( e \) : exponential function

2. **Theoretical premium**

The call premium and the put premium on the commodities futures contract and index are expressed as follows:

\[
C = e^{-RT} \cdot [N(d_1) - N(d_2)], \\
P = e^{-RT} \cdot [N(d_1) - 1] - E \cdot [N(d_2) - 1]
\]

Where

\[
d_1 = \frac{1}{\nu \cdot \sqrt{T}} \cdot \ln \left( \frac{U}{E} \right) + \frac{1}{2} \cdot \nu \cdot \sqrt{T}, \\
d_2 = d_1 - \nu \cdot \sqrt{T}
\]

In order to simplify the formula, the normal law is approximated with a fifth degree polynomial such as:

\[
P(d) = \frac{2}{\sqrt{2\pi}} \cdot e^{-\frac{d^2}{2}} \cdot (bx + cx^2 + fx^3 + gx^4 + ix^5)
\]

where:

\[
x = \frac{1}{(\alpha + a \cdot |d|)}
\]

and

\[
a = 0.231641900, \quad b = 0.319381530, \quad c = -0.356563782,
\]

\[
f = 1.781477937, \quad g = -1.821255978, \quad i = 1.330274429
\]
Then: \[
\begin{align*}
\text{if } d > 0 \quad N(d) &= 1 - P(d) \\
\text{if } d \leq 0 \quad N(d) &= P(d)
\end{align*}
\]

**Note:**

- **Option “theoretical” premium**
  For intra-day calculation, theoretical premiums are performed before risk array calculation. Therefore the formula is the same except that \( T_0 \) is used instead of \( T \).

- **Controlling via the intrinsic value of the option**
  If the Black 76 formula gives a theoretical premium that is inferior to the intrinsic value, the premium selected for determining the initial margin shall correspond to:
  - The intrinsic value if the exercise style of the option is “American” style
  - The theoretical premium if the exercise style of the option is “European” style.

- **Rounding off**
  The calculated risk arrays values related to SPAN\textsuperscript{®} scenarios are rounded off to two decimals.

### 3. Delta

The call delta and the put delta for the long rate and commodities futures contract and index are expressed as follows:

\[
\begin{align*}
\Delta_{Call} &= e^{-RT} \times N(d_1) \\
\Delta_{Put} &= e^{-RT} \times [N(d_1) - 1]
\end{align*}
\]

- **Rounding off**
  The calculated delta is rounded off to four decimals.
CHAPTER III

VALUATION FORMULA FOR EQUITY OPTIONS
As part of determining the initial margin required to cover the positions on equity stock options contracts, LCH.Clearnet SA uses a theoretical premium valuation formula of the type Cox Ross Rubinstein.

**ALGORITHM USED: THEORETICAL PREMIUM AND DELTA**

1. **Notation**

- \( S \): Index or settlement prices of the underlying contract of the option
- \( E \): Strike price of the option
- \( T_0 \): Time to maturity of the option (in fraction of year (base: 365 d or 366 d)).
- \( t \): Time to maturity of the option minus the “look-ahead time” of the option. The look-ahead time is average time per day (base: \((1/365) \) d or 0.00274). So it reflects the unit of time (in fraction of year) into the future from the current time \(( T = T_0 - \text{moveahead} \) or \( T = T_0 - 0.00274 \))
- \( \nu \): Yearly volatility of the underlying (expressed in %).
- \( R_t \): Financing rate corresponding to the life span of the options.
- \( n \): Number of iterations used in the model. \( n \) =30 iterations.
- \( h_s \): Amount of the dividend distributed to the iteration \( s \) (the model accepts 8 payments of dividends per year)
- \( T_0s \): number of days left before the payment of the dividend \( h_s \). The number is expressed in fraction of years (base: 365 d or 366 d)
- \( t_s \): number of days left after applying the look-ahead time and before the payment of the dividend \( h_s \). The number is expressed in fraction of years (base: 365 d or 366 d)

\[ \text{moveahead}T_0 \text{moveahead}T = \text{moveahead}T_0 \text{moveahead}T = \text{moveahead}T_0 - 0.00274 = \text{moveahead}T_0 - 0.00274 \]

- **Definition of intermediary data**

\( r = (1 + R_t)^{1/n} \) is the continuous interest rate

- **Definition of the “neutral risk” probability**

  - If no dividend is paid during the life span of the option, the ‘neutral risk’ probability is:

\[
q = \frac{r - d}{u - d}
\]

where \( u = e^{\sqrt{\text{nu}}} \), and \( d = \frac{1}{u} \)

  - If the occurrence \( s \) for the payment of the dividend \( h_s \) corresponds to the iteration \( i \), the ‘neutral risk’ probability becomes:

\[
q_{i,s} = q - \frac{u^{i-2j+2} \cdot S}{u - d} \cdot \ldots \cdot 1 \leq i \leq n - 1 \ldots \cdot 1 \leq j \leq i + 1
\]

where

- \( i = s \)
- \( h_s \) is the amount of the dividend distributed at the iteration \( s \).

The iteration \( s \) is the integer part of \( n \cdot \left( \frac{t_s}{T} \right) \).

**Note:** if the iteration \( i = s \), no dividend is paid (\( h_{i,s} \) =0). In that case, the ‘neutral risk’ probability is equal to \( q \).
**Valuation Formula for Equity Options**

**Dividend Management for Option Valuation:** in case there is not a full distribution of dividends detached over the complete option lifetime, the last known dividend value received for an option instrument will be used and duplicated until the maturity date of the option as often as possible from the last known detachment date according to a certain frequency type. Three types of frequency can be applied and are expressed in number of days: yearly (365 days), half-yearly (182 days), quarterly (91 days). This frequency type is defined at the combined commodity level in the margin parameters and follows the market rules (e.g. Paris securities generally detach a yearly dividend).

- $C_{i,j}$ (respectively $P_{i,j}$) is the price of the call (resp. put) at the iteration $i$ in the hypothesis $j$ considering the cost evolution of the underlying instrument.

## 2 Theoretical Premium

The theoretical premium for each scenario is performed with 30 iterations.

As the $n+1$ $C_{n,j}$ prices (respectively $P_{n,j}$) at the call month date (resp. put) corresponding to the iteration $n$ and to each hypotheses $j$ considering the cost evolution of the underlying instrument are known, the theoretical premium is calculated from the following recurrence formula after $n$ iterations, and differentiating the European and the American character of the option.

### 2.1 European Options (30 Iterations)

<table>
<thead>
<tr>
<th>Call</th>
<th>Put</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>At expiry,</strong></td>
<td><strong>At expiry,</strong></td>
</tr>
<tr>
<td>$C_{n,j} = \max \left{ \frac{n^2-j^2}{2.2} S \right} E$</td>
<td>$P_{n,j} = \max \left{ \frac{n^2-j^2}{2.2} S \right} E$</td>
</tr>
<tr>
<td>for $j=1,...,n+1$</td>
<td>for $j=1,...,n+1$</td>
</tr>
<tr>
<td>$C_{i,j} = \frac{q_{i,j} * C_{i+1,j} + \left( -q_{i,j} \right) C_{i+1,j+1}}{r}$</td>
<td>$P_{i,j} = \frac{q_{i,j} * P_{i+1,j} + \left( -q_{i,j} \right) P_{i+1,j+1}}{r}$</td>
</tr>
<tr>
<td>for $i = n-1, n-2, ..., 1, 0$ et $j = 1, 2, ..., i, i+1$</td>
<td>for $i = n-1, n-2, ..., 1, 0$ et $j = 1, 2, ..., i, i+1$</td>
</tr>
<tr>
<td><strong>Theoretical premium at the valuation moment</strong></td>
<td><strong>Theoretical premium at the valuation moment</strong></td>
</tr>
<tr>
<td>$C_{0,1} = \frac{q_{0,1} * C_{1,1} + \left( -q_{0,1} \right) C_{1,2}}{r}$</td>
<td>$P_{0,1} = \frac{q_{0,1} * P_{1,1} + \left( -q_{0,1} \right) P_{1,2}}{r}$</td>
</tr>
</tbody>
</table>

**Note:**

- **Option "theoretical" premium**
  For intra-day calculation, theoretical premiums are performed before risk array calculation. Therefore the formula is the same except that $T_0$ is used instead of $t$.

- **Rounding off**
  The calculated risk arrays values related to SPAN® scenarios are rounded off to two decimals.

### 2.2 American Options (30 Iterations)

<table>
<thead>
<tr>
<th>Call</th>
<th>Put</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>At expiry,</strong></td>
<td><strong>At expiry,</strong></td>
</tr>
</tbody>
</table>

VALUATION FORMULA FOR EQUITY OPTIONS

\[ C_{n,j} = \max \left\{ \left( n^2 j + S \right) \right\} \]
for \( j = 1, \ldots, n+1 \)

\[ P_{n,j} = \max \left\{ E - \left( n^2 j + S \right) \right\} \]
for \( j = 1, \ldots, n+1 \)

\[ C_{i,j} = \max \left\{ \frac{h_{i,j} \cdot C_{i+1,j} + \left( -q_{i,j} \cdot 2^j \cdot C_{i+1,j+1} \right)}{r} \right\} \]
for \( i = n-1, n-2, \ldots, 1, 0 \) and \( j = 1, 2, \ldots, i, i+1 \)

\[ P_{i,j} = \max \left\{ \frac{h_{i,j} \cdot C_{i+1,j} + \left( -q_{i,j} \cdot 2^j \cdot P_{i+1,j+1} \right)}{r} \right\} \]
for \( i = n-1, n-2, \ldots, 1, 0 \) and \( j = 1, 2, \ldots, i, i+1 \)

Theoretical premium at the valuation moment

\[ C_{0,1} = \max \left\{ \frac{h_{0,1} \cdot C_{1,1} + \left( -q_{0,1} \cdot 2^1 \cdot C_{1,2} \right)}{r} \right\} \]

\[ P_{0,1} = \max \left\{ \frac{h_{0,1} \cdot C_{1,1} + \left( -q_{0,1} \cdot 2^1 \cdot P_{1,2} \right)}{r} \right\} \]

Note:

- Option "theoretical" premium
  For intra-day calculation, theoretical premiums are performed before risk array calculation. Therefore the formula is the same except that \( T_0 \) is used instead of \( t \).

- Rounding off
  The calculated risk arrays values related to SPAN® scenarios are rounded off to two decimals.

3. Delta

The call delta and the put delta are calculated as follows:

<table>
<thead>
<tr>
<th>Call</th>
<th>Put</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta_{\text{Call}} = \frac{C}{x} ) when ( x ) is close to ( C )</td>
<td>( \Delta_{\text{Put}} = \frac{P}{x} ) when ( x ) is close to ( P )</td>
</tr>
</tbody>
</table>

- Controlling the price size
  The following check value is performed on the underlying price in order to adapt the accuracy of the delta calculation:

  By default, the value of the parameter \( x \) is set to 0.1 without valuation. To be consistent, this parameter must be valued with the option \( \text{CVF} \) (as it is done for premiums and market price).

  If \( y \neq S \) then \( x = y \neq S \) Else \( x \) is kept unchanged.

  By default, the value of \( y \) is set to 0.1 without valuation. To be consistent, this parameter must be valued with the option \( \text{CVF} \) (as it is done for premiums and market price).

- Rounding off
  The calculated delta is rounded off to four decimals.
CHAPTER IV

VALUATION FORMULA FOR CURRENCY OPTIONS
For determining the initial margin required to cover the positions on options on currency futures contracts, LCH.Clearnet SA uses a theoretical premium valuation formula of the type Garman kohlhagen.

ALGORITHM USED: THEORETICAL PREMIUM AND DELTA

1. Notation

The algorithm requires the following data:

- **U**: Price of the underlying futures contract of the option
- **E**: Strike price of the option
- **T**<sub>0</sub>: Time to maturity of the option (in fraction of year (base: 365 d or 366 d)).
- **T**: A time to maturity minus the "look-ahead time" of the option. The look-ahead time is average time per day (base: (1/365) d or 0.00274). So it reflects the unit of time (in fraction of year) into the future from the current time ($T = T_0 - \text{moveahead}$ or $T = T_0 - 0.00274$)
- **v**: Yearly volatility of the underlying (expressed in %).
- **R<sub>d</sub>**: Yearly continuous financing domestic rate on the life span of the option
- **R<sub>f</sub>**: Yearly continuous financing foreign rate on the life span of the option
- **N(x)**: Distribution function of the normal law
- **Ln**: Napierian logarithm
- **e**: Exponential function

2. Theoretical premium

The call premium and the put premium on the currency futures contract are expressed as follows:

$$C = U * e^{-R_f * T} * N(d_1) - E * e^{-R_d * T} * N(d_2)$$

$$P = U * e^{-R_f * T} * \left[ N(d_1) - 1 \right] - E * e^{-R_d * T} * \left[ N(d_2) - 1 \right]$$

Where

$$d_1 = \frac{1}{v * \sqrt{T}} * \ln \left( \frac{U * e^{-R_f * T}}{E} \right) + \frac{1}{2} * v * \sqrt{T}$$

$$d_2 = d_1 - v * \sqrt{T}$$

In order to simplify the formula, the normal law is approximated with a fifth degree polynomial such as:

$$P(d) = \frac{2}{\sqrt{2\pi}} * e^{-\frac{d^2}{2}} * (bx + cx^2 + fx^3 + gx^4 + hx^5)$$

where:

- **Initial Margin calculation on derivative markets**: Option valuation methods
- **LCH.Clearnet SA**


\[
x = \frac{1}{1 + a \times \vert d \vert}
\]

and

\[
\begin{align*}
a &= 0.231641900 & b &= 0.319381530 & c &= -0.356563782 \\
f &= 1.781477937 & g &= -1.821255978 & i &= 1.330274429
\end{align*}
\]

Then:

- if \( d > 0 \) \( N(d) = 1 - P(d) \)
- if \( d \leq 0 \) \( N(d) = P(d) \)

Note:

- Option “theoretical” premium
  For intra-day calculation, theoretical premiums are performed before risk array calculation. Therefore the formula is the same except that \( T_0 \) is used instead of \( T \).

- Rounding off
  The calculated risk arrays values related to SPAN® scenarios are rounded off to two decimals.

3. Delta

The call delta and the put delta for the currency futures contract are expressed as follows:

\[
\begin{align*}
\Delta_{Call} &= e^{-R_fT} \times \frac{N(d)}{1} \\
\Delta_{Put} &= e^{-R_fT} \times \frac{1}{1} - 1
\end{align*}
\]

- Rounding off
  The calculated delta is rounded off to four decimals.