INITIAL MARGIN CALCULATION ON DERIVATIVE MARKETS

OPTION VALUATION FORMULAS

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CONTENTS

CHAPTER I

VALUATION FORMULA FOR OPTIONS ON FUTURES AND INDICES

For determining the Initial Margin required to cover the positions on options on futures contracts (commodity for instance) and options on index, LCH SA uses a theoretical premium valuation formula of the Black 76 type.

THEORICAL PREMIUM AND DELTA FORMULAS

NOTATION

The algorithm requires the following data:

U : Price of the underlying futures contract of the option
E : Strike price of the option
T : Life span of the option (in fraction of year (base: 365 d or 366 d))
ν : Yearly volatility of the underlying (expressed in %)
R : Yearly continuous financing rate on the life span of the option ($r = Ln(1+R)$ where R is the yearly financing rate)
N(x) : Distribution function of the normal law
Ln : Neperian logarithm
e : Exponential function

THEORETICAL PREMIUM

The call premium and the put premium on commodity future contracts and index are expressed as follows:

$$C = e^{-r*T} * [U * N(d_1) - E * N(d_2)]$$

$$P = e^{-r*T} * [U * (N(d_1) - 1) - E * (N(d_2) - 1)]$$

Where,

$$d_1 = \frac{1}{v * \sqrt{T}} * Ln\left(\frac{U}{E}\right) + \frac{1}{2} * v * \sqrt{T} \qquad d_2 = d_1 - v * \sqrt{T}$$

The call premium and the put premium must be calculated as follows:

U = 100 – U and E = 100 – E for determining parameter d_1 (and therefore d_2).

$$C = e^{-r*T} * [(100 - U) * (N(d_1) - 1) - (100 - E) * (N(d_2) - 1)]$$

$$P = e^{-r*T} * [(100 - U) * N(d_1) - (100 - E) * N(d_2)]$$

In order to simplify the formula, the normal law is approximated with a fifth degree polynomial such as:

$$P(d) = \frac{1}{\sqrt{2\pi}} * e^{\frac{-d^2}{2}} * (bx + cx^2 + fx^3 + gx^4 + ix^5)$$

Where,

$$x = \frac{1}{\left(1 + a * |d|\right)}$$

And, a = 0.231641900 b = 0.319381530 c = -0.356563782
f = 1.781477937 g = -1.821255978 i = 1.330274429

Then, if d > 0 N(d) = 1 – P(d)
if d ≤ 0 N(d) = P(d)

<u>Controlling via the intrinsic value of the option</u>
If the Black 76 formula gives a theoretical premium that is inferior to the intrinsic value, the premium selected for determining the initial margin shall correspond to the intrinsic value.

DELTA

The call delta and the put delta commodities futures contract and index are expressed as follows:

$$\Delta_{Call} = e^{-rT} * N(d_1) \qquad \Delta_{Put} = e^{-rT} * [N(d_1) - 1]$$

Similarly:

The call delta and the put delta must be calculated with the following formulae:

$$\Delta_{Call} = -e^{-rT} * [N(d_1) - 1] \qquad \Delta_{Put} = -e^{-rT} * N(d_1)$$

ROUND-UP RULES

The calculated Risk Arrays values related to SPAN® scenarios are rounded to two decimals.
The calculated Delta is rounded to four decimals.

CHAPTER II

VALUATION FORMULA FOR EQUITY OPTIONS

As part of determining the Initial Margin required to cover the positions on equity options, LCH SA uses a theoretical premium valuation formula of the Cox Ross Rubinstein type.

THEORICAL PREMIUM AND DELTA FORMULAS

NOTATION

S : Index or settlement prices of the underlying contract of the option
K : Strike price of the option
t : Life span of the option (in fraction of year (base: 365 d or 366 d))
ν : Yearly volatility of the underlying (expressed in %).
R_t : Financing rate corresponding to the life span of the options. It is chosen among the following rate maturities: 1 month, 3 months, 6 months, 9 months, 12 months and 2 years.
n : Number of iterations used in the model
Q_s : Dividend value
q_s : Dividend payment date in years (ACT/365, between payment date and clearing date).

- DEFINITION OF INTERMEDIARY DATA:

- $r = (1 + R_t)^{\frac{t}{n}}$ is the continuous interest rate

- DEFINITION OF THE "NEUTRAL RISK" PROBABILITY.

- The 'neutral risk' probability is:

$$q = \frac{r-d}{u-d}$$

where $u = e^{v\sqrt{\frac{t}{n}}}$, and $d = \frac{1}{u}$

- DEFINITION OF THE TREE

The CRR probability tree is built starting from $S_{0,1} = S$

For each iteration i, the underlying forward value is:

$$S_{i+1,j} = \begin{pmatrix} S_{i,j-1} * d \\ or \\ S_{i,j} * u \end{pmatrix}$$

$$t_i = t \times \frac{i}{n}$$

Let us denote $C_{i,j}$ (respectively $P_{i,j}$) the price of the call (respectively put) at the iteration i in the hypothesis j considering the cost evolution of the underlying instrument.

We assume that, there is not a doubt regarding the payment of the dividend at date q_s.

The probability p for the detachment of the dividend is 1.

We denote m_i the multiplier that will be used for all $S_{i,?}$, the nodes at iteration i.

$$m_i = 1 - \sum_{q_s < t_i} \frac{Q_s \times (1+r)^{-q_s}}{S_{0,1}}$$

When a detachment of dividend happens, at the iteration i=s, the $S_{i,j}$ are replaced by $S'_{i,j} = S_{i,j} \times m_i$ for the current nodes.

Dividend management for option valuation
In case there is not a full distribution of dividends detached over the complete option lifetime, the last known dividend value received for an option instrument will be used and duplicated until the expiration date of the option as often as possible from the last known detachment date according to a certain frequency type. Three types of frequency can be applied and are expressed in number of days: yearly (365 days), half-yearly (182 days), quarterly (91 days). This frequency type is defined at the combined commodity level in the margin parameters and follows the market rules (e.g. Paris securities generally detach a yearly dividend)

THEORETICAL PREMIUM

The theoretical premium selected is the mathematical average of two theoretical premiums calculated with this model; one would have n iterations, and the other would have n+1 iterations.

As the n+1 $C_{n,j}$ prices (respectively $P_{n,j}$) at the call month date (respectively put) corresponding to the iteration n and to each hypotheses j considering the cost evolution of the underlying instrument are known, the theoretical premium is calculated from the following recurrence formula after n iterations, and differentiating the European and the American character of the option.

- EUROPEAN OPTIONS (30/31 ITERATIONS)

Call	**Put**
At expiry, $C_{n,j} = max\{0;\ S'_{n,j} - K\}$ for j=1,…,n+1	At expiry, $P_{n,j} = max\{0;\ K - S'_{n,j}\}$ for j=1,…,n+1
$C_{i,j} = \frac{q \cdot C_{i+1,j} + (1-q) \cdot C_{i+1,j+1}}{r}$ for i = n-1, n-2,…,1, 0 et j = 1, 2,…, i, i+1	$P_{i,j} = \frac{q \cdot P_{i+1,j} + (1-q) \cdot P_{i+1,j+1}}{r}$ for i = n-1, n-2,…,1, 0 et j = 1, 2,…, i, i+1
Theoretical premium at the valuation moment $C = \frac{q \cdot C_{1,1} + (1-q) \cdot C_{1,2}}{r}$	Theoretical premium at the valuation moment $P = \frac{q \cdot P_{1,1} + (1-q) \cdot P_{1,2}}{r}$

- AMERICAN OPTIONS (30/31 ITERATIONS)

Call	Put
At expiry, $$C_{n,j} = max\{0;\ S'_{n,j} - K\}$$ for j=1,…,n+1	At expiry, $$P_{n,j} = max\{0;\ K - S'_{n,j}\}$$ for j=1,…,n+1
$$C_{i,j} = max\left(\frac{q \cdot C_{i+1,j} + (1-q) \cdot C_{i+1,j+1}}{r}; S'_{i,j} - K\right)$$ for i = n-1, n-2,…,1, 0 et j = 1, 2,…, i, i+1	$$P_{i,j} = max\left(\frac{q \cdot P_{i+1,j} + (1-q) \cdot P_{i+1,j+1}}{r}; K - S'_{i,j}\right)$$ for i = n-1, n-2,…,1, 0 et j = 1, 2,…, i, i+1
Theoretical premium at the valuation moment $$C = max\left(\frac{q \cdot C_{1,1} + (1-q) \cdot C_{1,2}}{r};\ S'_{1,0} - K\right)$$	Theoretical premium at the valuation moment $$P = max\left(\frac{q \cdot P_{1,1} + (1-q) \cdot P_{1,2}}{r};\ K - S'_{1,0}\right)$$

DELTA

The call delta and the put delta are calculated as follows:

Call	Put
$$\Delta_{Call} = \frac{C(S+x) - C(S-x)}{2 * x}$$	$$\Delta_{Put} = \frac{P(S+x) - P(S-x)}{2 * x}$$

- CONTROLLING THE PRICE SIZE.

  The following check value is performed on the underlying price in order to adapt the accuracy of the delta calculation:

- If $y * S < x$, then $x = y * S$. Else x is kept unchanged.

  By default, the value of y is set to 0.1 and the value of the parameter x is set to 0.1.

ROUND-UP RULES

The calculated Risk Arrays values related to SPAN® scenarios are rounded off to two decimals.
The calculated Delta is rounded off to four decimals.

CHAPTER III

VALUATION FORMULA FOR CURRENCY OPTIONS

For determining the Initial Margin required to cover the positions on options on currency contracts, LCH SA uses a theoretical premium valuation formula of the Garman-Kohlhagen type.

THEORICAL PREMIUM AND DELTA FORMULAS

NOTATION

The algorithm requires the following data:

U : Price of the underlying futures contract of the option
K : Strike price of the option
T : Life span of the option (in fraction of year (base: 365 d or 366 d))
ν : Yearly volatility of the underlying (expressed in %).
R_d : Yearly continuous financing domestic rate on the life span of the option ($r_d = Ln(1+R_d)$ where R_d is the yearly financing domestic rate)
R_f : Yearly continuous financing foreign rate on the life span of the option ($r_f = Ln(1+R_f)$ where R_f is the yearly financing foreign rate)
N(x) : Distribution function of the normal law
Ln : Neperian logarithm
e : Exponential function

THEORETICAL PREMIUM

The call premium and the put premium on the currency futures contract are expressed as follows:

$$C = U * e^{-r_f * T} * N(d_1) - K * e^{-r_d * T} * N(d_2)$$

$$P = U * e^{-r_f * T} * \left(N(d_1) - 1\right) - K * e^{-r_d * T} * \left(N(d_2) - 1\right)$$

Where

$$d_1 = \frac{1}{v * \sqrt{T}} * Ln\left(\frac{S * e^{(r_d - r_f) * T}}{K}\right) + \frac{1}{2} * v * \sqrt{T} \qquad d_2 = d_1 - v * \sqrt{T}$$

In order to simplify the formula, the normal law is approximated with a fifth degree polynomial such as:

$$P(d) = \frac{1}{\sqrt{2\pi}} * e^{\frac{-d^2}{2}} * \left(bx + cx^2 + fx^3 + gx^4 + ix^5\right)$$

Where,

$$x = \frac{1}{\left(1 + a * |d|\right)}$$

And,	a = 0.231641900	b = 0.319381530	c = -0.356563782
	f = 1.781477937	g = -1.821255978	i = 1.330274429

Then, if $d > 0$ $N(d) = 1 - P(d)$
if $d \leq 0$ $N(d) = P(d)$

DELTA

The call Delta and the put Delta for the currency futures contract are expressed as follows:

$$\Delta_{Call} = e^{-r_f T} * N(d_1) \qquad \Delta_{Put} = e^{-r_f T} * [N(d_1) - 1]$$

ROUND-UP RULES

The calculated risk arrays values related to SPAN® scenarios are rounded off to two decimals.
The calculated delta is rounded off to four decimals.